Designing 2-D State Observers for Delayed Discrete Systems using LMIs

AMAL HADER[1], CHAKIR EL-KASRI[2] and MOHAMMED ALFIDI[1]

[1] Sidi Mohamed Ben Abdellah University, National School of Applied Sciences, My Abdallah Avenue Km. 5 Imouzzer Road, Fez, Morocco,
hader.amal@gmail.com, and alfidi_mohammed@yahoo.fr

[2] Sidi Mohamed Ben Abdellah University, Polydisciplinary Faculty, Oujda road, Taza, Morocco,
chakir.elkasri@usmba.ac.ma

Abstract. This paper addresses the state observer problem for discrete two-dimensional (2-D) systems with delays described by the Roesser model. The main objective of the design is to ensure asymptotic stability by designing a 2-D observer. It sounds like the paper is proposing a new method for designing a 2-D observer for a given system. The method is based on two key concepts: the Lyapunov function and the linear matrix inequalities (LMIs) formalism. The example is utilized to showcase how the method can be practically applied to a given system and to evaluate the observer's performance.

Keywords: Observer; Stability; Linear Matrix Inequality (LMI); Roesser Model; 2-D discrete systems.

1 Introduction

It is interesting to see that the Fornasini-Marchesini local state-space second model [9] and the Roesser model [16] have been extensively studied in various domains. These models are widely used in control theory and signal processing applications due to their ability to describe systems with time-varying coefficients and time-delayed inputs.

In the domain of controller design, researchers such as [3, 17] have studied the use of these models in designing controllers for various applications. The Roesser model has been shown to be particularly useful for designing robust controllers that can handle uncertainties and disturbances.

In the domain of filter design, researchers such as [8, 14, 10] have studied the use of these models in designing various types of filters such as Kalman filters and H-infinity filters. These filters are widely used in signal processing applications to estimate the states of a system and remove noise from signals.

Stability analysis and stabilization of these models have been addressed by researchers such as [4–6, 2, 12]. These studies have focused on developing techniques to ensure the stability of the system under various conditions and designing controllers that can stabilize the system.

Finally, observer design without delay has been addressed by researchers such as [1, 11]. Observers are used in control theory to estimate the states of a system when only limited measurements are available. These studies have focused on designing observers that can estimate the states of the system accurately and in real-time.

This paper deals with the problem of observing the state vector of a 2-D discrete system with delays, which is described by the Roesser model. The goal is to design a delayed 2-D state observer that estimates the system's state variables accurately.

To achieve this goal, the paper proposes a new sufficient condition for the design of the observer. The condition is based on two mathematical tools: the linear matrix inequality (LMI) method and the Lyapunov theory.

The LMI method is a powerful tool for solving optimization problems involving linear matrix inequalities. It provides a way to find feasible solutions to a wide range of problems, including control, estimation, and optimization. In this paper, the LMI method is used to find a set of observer gain matrices that satisfy the proposed condition.

Notation: The paper introduces notation conventions that will be used throughout the document. The n-dimensional real Euclidean space is represented as $\mathbb{R}^n$, while $\mathbb{R}^{n\times m}$ denotes the set of matrices that are n by m. When referring to real symmetric matrices M, the notation $M > 0$ indicates that the matrix is positive definite. The n-dimensional identity matrix is represented as I_n. The transpose of a matrix is indicated by the superscript "T", while block-diagonal matrices are represented as *diag*.... The notation $Symm(M)$ is used to represent $M + M^T$. In a symmetric matrix, the symmetric term takes the form: *, e.g. $\begin{bmatrix} M & N \\ * & X \end{bmatrix} = \begin{bmatrix} M & N \\ N^T & X \end{bmatrix}$. Unless explicitly stated, matrices are assumed to have compatible dimensions.

2 preliminary steps and problem formulation

The Roesser state space model [16] is used to define a 2-D discrete system with delays, which is presented in this section as follows:

$$(\Sigma):\begin{cases}\begin{bmatrix}\frac{\partial x^h(i,j)}{\partial i}\\ \frac{\partial x^v(i,j)}{\partial j}\end{bmatrix} &= \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix} + \begin{bmatrix}A_{d11} & A_{d12}\\ A_{d21} & A_{d22}\end{bmatrix}\begin{bmatrix}x^h(i-d_h,j)\\ x^v(i,j-d_v)\end{bmatrix}\\ &\quad + \begin{bmatrix}B_1\\ B_2\end{bmatrix}u(i,j)\\ y(i,j) &= \begin{bmatrix}C_1 & C_2\end{bmatrix}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}\end{cases} \tag{1}$$

where $x^h(i,j)\in\mathbb{R}^{n_h}$ and $x^v(i,j)\in\mathbb{R}^{n_v}$ are the horizontal and vertical state vectors, respectively, $u(i,j)\in\mathbb{R}^m$ is the input vector and $y(i,j)\in\mathbb{R}^l$ is the measured output vector. $A=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\in\mathbb{R}^{(n_h+n_v)\times(n_h+n_v)}$, $A_d=\begin{bmatrix}A_{d11} & A_{d12}\\ A_{d21} & A_{d22}\end{bmatrix}\in$

$\mathbb{R}^{(n_h+n_v)\times(n_h+n_v)}$, $\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \in \mathbb{R}^{(n_h+n_v)\times m}$ and $\begin{bmatrix} C_1 \; C_2 \end{bmatrix} \in \mathbb{R}^{l\times(n_h+n_v)}$ are real constant matrices of appropriate dimensions. d_h and d_v represent a horizontal and vertical delays, respectively.

$$d_{hM} \geq d_h(i) \geq d_{hm}, \quad d_{vM} \geq d_v(j) \geq d_{vm} \tag{2}$$

The concepts used in this paper are introduced after defining the boundary conditions of the 2-D delayed system (Σ) using d_{hm}, d_{vm}, d_{hM}, and d_{vM} as the lower and upper bounds of the delays d_h and d_v. These boundary conditions are specified as:

$$\begin{cases} x(k,j) = 0 \quad \forall\, j \geq 0 \;\&\; k = -d_h,\ -d_h+1,\ ...,0 \\ x(i,l) = 0 \quad \forall\, i \geq 0 \;\&\; l = -d_v,\ -d_v+1,\ ...,0 \end{cases} \tag{3}$$

Next, we will present the concepts that are utilized in this paper.

Definition 1. *[7]. The 2-D discrete-time linear system with state delay (Σ) is considered asymptotically stable for $u = 0$ and all bounded boundary conditions specified in (3) if:*

$$\lim_{r\to\infty} \chi_r = 0 \tag{4}$$

where,

$$\chi_r = sup\{\| x(i,j) \|: r = i+j, i, j \geq 1\} \tag{5}$$

In the absence of input and output vectors in system (1), we can describe the corresponding 2-D state-delayed discrete system (Σ_0) as follows:

$$\begin{aligned} (\Sigma_0) : \begin{bmatrix} \frac{\partial x^h(i,j)}{\partial i} \\ \frac{\partial x^v(i,j)}{\partial j} \end{bmatrix} &= \begin{bmatrix} A_{11} \; A_{12} \\ A_{21} \; A_{22} \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \\ &+ \begin{bmatrix} A_{d11} \; A_{d12} \\ A_{d21} \; A_{d22} \end{bmatrix} \begin{bmatrix} x^h(i-d_h,j) \\ x^v(i,j-d_v) \end{bmatrix} \end{aligned} \tag{6}$$

The following lemma presents a sufficient condition for the asymptotic stability of the 2-D system with delay (1) in the absence of an input vector, i.e., when $u(i,j) = 0$.

Lemma 1. *[15]: A 2-D system, as described by Equation (6), is quadratically stable for any delay $0 < d \leq \bar{d}$ if there exist matrices of the form: $P = \begin{bmatrix} P_h & 0 \\ 0 & P_v \end{bmatrix} > 0$, $M = \begin{bmatrix} M_h & 0 \\ 0 & M_v \end{bmatrix} > 0$ and $Q = \begin{bmatrix} Q_h & 0 \\ 0 & Q_v \end{bmatrix} > 0$ and X, Y so that the following LMIs apply:*

$$\begin{bmatrix} -P + dX + Symm(Y) + M & * & * & * \\ -Y^T & -M & * & * \\ PA & PA_d & -P & * \\ dQ(A-I) & dQA_d & 0 & -dQ \end{bmatrix} < 0 \tag{7}$$

$$\begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \geq 0 \tag{8}$$

3 Designing a two-dimensional observer with delays.

This section presents a constructive method for designing a 2-D state observer for a 2-D discrete system (Σ).

Our goal is to construct a full-order observer that can estimate the state vector $x(i,j)$ completely, even in the presence of delays where the state vector cannot be fully measured or inferred from the outputs. The structure of this observer can be represented as follows:

$$(\Sigma_o):\begin{cases}\begin{bmatrix}\frac{\partial x_o^h(i,j)}{\partial i}\\ \frac{\partial x_o^v(i,j)}{\partial j}\end{bmatrix} = \begin{bmatrix}N_{11} & N_{12}\\ N_{21} & N_{22}\end{bmatrix}\begin{bmatrix}x_o^h(i,j)\\ x_o^v(i,j)\end{bmatrix} + \begin{bmatrix}N_{d11} & N_{d12}\\ N_{d21} & N_{d22}\end{bmatrix}\begin{bmatrix}x_o^h(i-d_h,j)\\ x_o^v(i,j-d_v)\end{bmatrix}\\ \qquad + \begin{bmatrix}L_1\\ L_2\end{bmatrix} y(i,j) + \begin{bmatrix}M_1\\ M_2\end{bmatrix} u(i,j)\\ \tilde{x}(i,j) \qquad = x_o(i,j) + Ey(i,j)\end{cases} \tag{9}$$

where $x_o^h(i,j) \in \mathbb{R}^{n_h}$ and $x_o^v(i,j) \in \mathbb{R}^{n_v}$ are the horizontal and vertical state vectors of the 2-D observer, respectively, $\hat{x}(i,j) \in \mathbb{R}^{n_h+n_v}$ is the estimate of $x_o(i,j)$, $u(i,j) \in \mathbb{R}^m$ is the input vector and $y(i,j) \in \mathbb{R}^l$ is the measured output vector. The matrices $N \in \mathbb{R}^{(n_h+n_v)\times(n_h+n_v)}$, $N_d \in \mathbb{R}^{(n_h+n_v)\times(n_h+n_v)}$, $M \in \mathbb{R}^{(n_h+n_v)\times m}$, $L \in \mathbb{R}^{l\times(n_h+n_v)}$ and $E \in \mathbb{R}^{(n_h+n_v)\times l}$ must be chosen in such a way that the observation error asymptotically vanishes to zero.

The boundary conditions given in equations (2) are identical for the full-order 2-D observer system (9).

This section presents a new constructive approach for designing a 2-D state observer with delay, which guarantees the asymptotic convergence of the estimated state vector $\tilde{x}(i,j)$ to the true state vector $x(i,j)$ by utilizing the findings presented in the preceding sections. Specifically, we analyze the dynamics of the estimation error, which is defined as follows:

$$\begin{bmatrix}e^h(i,j)\\ e^v(i,j)\end{bmatrix} = \begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix} - \begin{bmatrix}\tilde{x}^h(i,j)\\ \tilde{x}^v(i,j)\end{bmatrix} \tag{10}$$

Remark 1. For the observer to be asymptotic, the estimation error must approach zero as (i,j) increases. However, since the state is inaccessible at $(i,j) = (0,0)$, we cannot generally choose $x(0,0) = \tilde{x}(0,0)$ and thus the estimation error $e(0,0) \neq 0$. In order to ensure the asymptotic convergence of the estimation error $e(i,j) \to 0$ as $(i,j) \to +\infty$, and to ensure proper operation of the observer, the matrices N, N_d, L, E, and M defined in (9) must be obtained using a wise LMI-based approach..

We present a novel sufficient condition for determining the matrices of the 2-D observer.

Theorem 1. *To achieve asymptotic estimation of the state vector using the 2-D state observer with delay (9), it is sufficient for the following conditions to hold:*

[label=)]if there exist matices the form $P = \begin{bmatrix} P_h & 0 \\ 0 & P_v \end{bmatrix} > 0$, $M = \begin{bmatrix} M_h & 0 \\ 0 & M_v \end{bmatrix} > 0$ *and* $Q = \begin{bmatrix} Q_h & 0 \\ 0 & Q_v \end{bmatrix} > 0$ *and matrices* W, W_q, F *and* F_q *such that the following LMIs hold:*

$$\begin{bmatrix} -P + dX + Symm(Y) + R & * & * & * \\ -Y^T & -R & * & * \\ PA - WCA - FC & PA_d - WCA_d + F_dC & -P & * \\ d(Q(A-I) - W_qCA - F_qC) & d(QA_d - W_qCA_d + F_{qd}C) & 0 & -dQ \end{bmatrix} < 0 \quad (11)$$

$$\begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \geq 0 \quad (12)$$

$$\begin{aligned} N &= A - ECA - KC, \quad K = L - NE \\ N_d &= A_d - ECA_d - K_dC, \quad K_d = N_dE, \\ M &= (I_n - EC)B. \end{aligned}$$

2. *Proof.* From the dynamics of the state reconstruction error given in (10), we can be written

$$e(i,j) = x(i,j) - \tilde{x}(i,j) \quad (13)$$

with

$$e(i,j) = \begin{bmatrix} e^h(i,j) \\ e^v(i,j) \end{bmatrix}, x(i,j) = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \text{ and } \tilde{x}(i,j) = \begin{bmatrix} \tilde{x}^h(i,j) \\ \tilde{x}^v(i,j) \end{bmatrix} \quad (14)$$

The 2-D discrete system with delays ($\tilde{\Sigma}$) is said to be an asymptotic observer for 2-D delayed system (Σ) if $\lim_{(i+j)\to+\infty} \| e(i,j) \| \to 0$ for any boundary conditions given in (2).

By (9) and (13), we have

$$e(i,j) = \Phi x(i,j) - x_o(i,j) \quad (15)$$

where $\Phi = I_n - EC$, then,

$$x_o(i,j) = \Phi x(i,j) - e(i,j) \quad (16)$$

Replacing $x_o(i,j)$ by $\Phi x(i,j) - e(i,j)$, $y(i,j)$ by $Cx(i,j)$.

Then, the temporal evolution of this estimation error can be expressed in the following way.

$$\begin{aligned} \begin{bmatrix} \frac{\partial e^h(i,j)}{\partial i} \\ \frac{\partial e^v(i,j)}{\partial j} \end{bmatrix} &= N \begin{bmatrix} e^h(i,j) \\ e^v(i,j) \end{bmatrix} + N_d \begin{bmatrix} e^h(i-d_h,j) \\ e^v(i,j-d_v) \end{bmatrix} \\ &+ (\Phi A - N\Phi - LC) \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \\ &+ (\Phi A_d - N_d\Phi) \begin{bmatrix} x^h(i-d_h,j) \\ x^v(i,j-d_v) \end{bmatrix} \\ &+ (\Phi B - M)\, u(i,j) \end{aligned} \quad (17)$$

Consequently, the 2-D error system converges asymptotically to zero when the following conditions are met:

$$\begin{bmatrix} \frac{\partial e^h(i,j)}{\partial i} \\ \frac{\partial e^v(i,j)}{\partial j} \end{bmatrix} = N \begin{bmatrix} e^h(i,j) \\ e^v(i,j) \end{bmatrix} + N_d \begin{bmatrix} e^h(i-d_h,j) \\ e^v(i,j-d_v) \end{bmatrix} \tag{18}$$

is asymptotically stable and the matrices M, N, N_d and L are chosen to satisfy the following conditions:

$$\begin{cases} \Phi A - N\Phi - LC = 0 \\ \Phi A_d - N_d\Phi \qquad = 0 \\ \Phi B - M \qquad\quad = 0 \end{cases} \tag{19}$$

then, we have that

$$\begin{cases} LC - \Phi A + N\Phi = 0 \Rightarrow N(I_n - EC) + LC - \Phi A = 0 \\ \Phi A_d - Nd\Phi = 0 \qquad \Rightarrow (I_n - EC)A_d - N_d(I_n - EC) = 0 \\ \Phi B - M = 0 \qquad\quad \Rightarrow (I_n - EC)B - M = 0 \end{cases} \tag{20}$$

therefore, the 2-D observer matrices are given by:

$$\begin{cases} N = A - ECA - KC \\ N_d = A_d - ECA_d + K_dC \\ M = (I_n - EC)B \end{cases} \tag{21}$$

with

$$\begin{aligned} K \ &= L - NE \\ K_d &= N_dE \end{aligned} \tag{22}$$

and

$$L = K + NE \tag{23}$$

According to (23), one should first confirm that one can find the matrices E, N, and K if the observation error (17) is asymptotically stable and the three conditions b), c), and d) of Theorem 1 are verified, then one searches for the matrix L.

Using the information provided in lemma 1, we can conclude that the 2-D error system described in equation (18) will be asymptotically stable for any delay d_h and d_v satisfying $0 < d_h \leq \bar{d}_h$ and $0 < d_v \leq \bar{d}_v$, if there exist matrices of the form: $P = \begin{bmatrix} P_h & 0 \\ 0 & P_v \end{bmatrix} > 0$, $M = \begin{bmatrix} M_h & 0 \\ 0 & M_v \end{bmatrix} > 0$ and $Q = \begin{bmatrix} Q_h & 0 \\ 0 & Q_v \end{bmatrix} > 0$ and X, Y such that the following LMIs hold:

$$\begin{bmatrix} -P + dX + Symm(Y) + M & * & * & * \\ -Y^T & -M & * & * \\ PN & PN_d & -P & * \\ dQ(N-I) & dQN_d & 0 & -dQ \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \geq 0 \tag{25}$$

From the equation (21) we have

$$\begin{cases} PN = PA - PECA - PKC \\ PN_d = PA_d - PECA_d + PK_dC \\ ZN = ZA - ZECA - ZKC \\ ZN_d = ZA_d - ZECA_d + ZK_dC \end{cases} \tag{26}$$

which leads to unavoidable bilinearities in (24), that can be solved by using the change of variables defined by

$$\begin{cases} W = PE, \\ F = PK, \\ F_d = PK_d, \\ W_q = QE, \\ F_q = QK, \\ F_{qd} = QK_d, \end{cases} \tag{27}$$

As equation (11) can be derived from the bilinear matrix inequality (24), the matrices for the 2-D state observer can be obtained by means of the following iterative method.

Algorithm

- **Step1.** Find a set of matices the form $P = \begin{bmatrix} P_h & 0 \\ 0 & P_v \end{bmatrix} > 0$, $M = \begin{bmatrix} M_h & 0 \\ 0 & M_v \end{bmatrix} > 0$ and $Q = \begin{bmatrix} Q_h & 0 \\ 0 & Q_v \end{bmatrix} > 0$ and matrices W, W_q, F and F_q, which satisfyies (11).
- **Step2.** Compute matrices K, K_d and E using (27), then:

$$\begin{aligned} E &= P^{-1}X_1, \\ K &= P^{-1}X_2 \\ K_d &= P^{-1}X_3 \end{aligned} \tag{28}$$

- **Step3.** Compute matrices N and N_d given in (21).
- **Step4.** Compute matrices L in (23)

this completes the proof of Theorem1.

4 Numerical example

In this example, we study a 2-D discrete system with delay in the form (1), described by the following parameters [13]:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \; A_{d1} = \begin{bmatrix} 0.1 & 1.5 \\ 0.1 & 0.3 \end{bmatrix}, \; B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \; C = \begin{bmatrix} 2 & 1 \end{bmatrix}. \tag{29}$$

Our goal is to reconstruct the internal state of the 2-D system with delays given by equation (1). To accomplish this, we require the estimated states to converge asymptotically to the true values, which will enable us to build a 2-D state observer. More precisely, we aim for the horizontal $e^h(i,j)$ and vertical $e^v(i,j)$ estimation errors to converge to zero as $i, j \to +\infty$, respectively.

To design a 2-D asymptotic state observer, we will apply Theorem 1, and verify that the LMI condition given in (11) is feasible. Once this is established, we can use the algorithm to obtain the matrices for the 2-D observer with delays, which are as follows:

$$\begin{aligned} &N = \begin{bmatrix} 0.3367 & 0.3007 \\ 0.6389 & 0.6174 \end{bmatrix}, N_d = \begin{bmatrix} -0.0047 & -0.0022 \\ -0.0121 & -0.0096 \end{bmatrix}, L = \begin{bmatrix} -0.0173 \\ -0.0455 \end{bmatrix}, \\ &M = \begin{bmatrix} 0.5000 \\ 1.0000 \end{bmatrix}, \qquad E = \begin{bmatrix} 0.3435 \\ -0.2986 \end{bmatrix}. \end{aligned} \tag{30}$$

Figures 1 and 2 show the trajectories of the estimation error system $e^h(i,j)$ and $e^v(i,j)$, respectively, corresponding to random initial boundary conditions. As $i+j \to \infty$, all trajectories asymptotically converge to zero, indicating that the 2-D state observer with delays is indeed asymptotically stable.

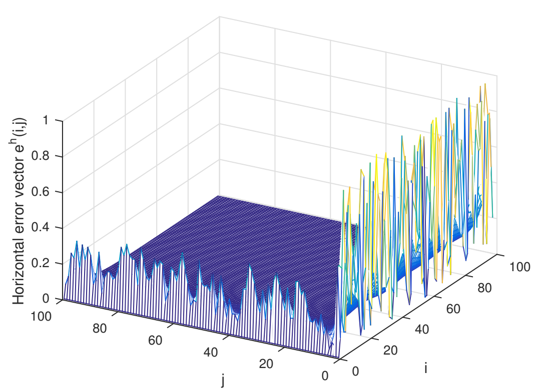

Fig. 1: Horizontal error $e^h(i,j)$

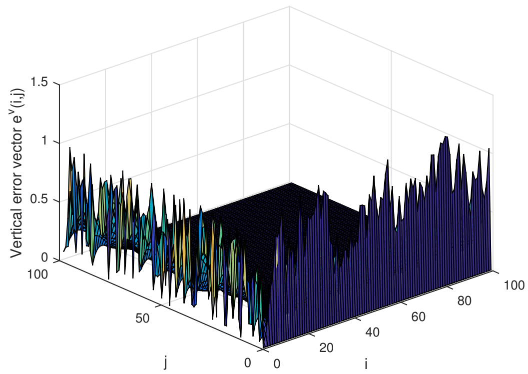

Fig. 2: Vertical error $e^v(i,j)$

5 Conclusion

Overall, the proposed method offers a new approach to designing 2-D state observers with delays that can be applied in various fields, such as image and signal processing, control, and communication systems. Additionally, the use of LMI conditions allows for efficient and robust design of the observer, and the simulation results demonstrate the effectiveness of the proposed method. Future work can focus on applying the proposed method to more complex systems and comparing its performance with other state observer designs.

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